# Mechanics of Vertical Moving Fluidized Systems

## II. Application to Countercurrent Operation

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**Experimental data are presented for two different methods of operating vertical moving fluidized systems of glass spheres and water: free countercurrent fluidization and bottom-restrained nonfed, or batch, fluidization. Data for a lead-shot–and–water system are also reported. A comparison is made of these data with a generalized theoretical analysis of ideal fluidized systems. The validity of a single characteristic holdup vs. slip-velocity relationship is illustrated, and consideration is also given to the prediction of flooding in the free countercurrent systems.**

Fluidized systems form an important area of fluid mechanics which is of primary concern to the chemical engineer. They are widely used in bringing two phases continuously into contact in order to permit mass or heat transfer. Subdivision of one phase into particles, bubbles, or droplets to carry out phase contacting is probably more common than the other two available methods: bulk or film contacting.

A fluidized system may be any device in which a mass of particles, whether solid, liquid, or gas, is suspended in a continuously moving fluid because of the motion of the fluid between the particles. When a mixture of particles and fluid is moved through a tower in a continuous stream, the area of contact between the phases available for mass or heat exchange depends upon the feed rate of each to the tower. In addition there is a dependence upon the size of the particles and the physical properties of particles and continuous fluid. The contact area is in turn determined directly by the particle holdup within the tower or its complement, the fractional voidage. The relation between void fraction and feed rates for given particles and fluid is a basic property of the system. The pressure gradient over the contact zone is, for example, fixed by the fractional holdup and the density difference between particles and fluid; it may be calculated if the void fraction is known.

Quantitative knowledge of the relationships between these factors is therefore fundamental to the rational treatment of any rate process within a fluidization zone and for the design of many phase-contacting operations in which one phase appears as a mass of particles. In a previous publication (*1*) a basic theory of vertical moving fluidized systems was presented which attempted to develop these necessary relationships. The basic premise of this theoretical development was the assumption that a single, fundamental relationship exists between the void fraction and the slip velocity for the particles and fluid. This relationship was assumed to be valid irrespective of the direction of flow of fluid and of particles in relation to each other and to the walls. It was further shown that the void fraction for any moving fluidized system could be predicted for all combinations of particle and fluid flow rates if data from a single batch fluidization experiment were available. The present paper reports experimental data for free countercurrent operation of a glass-spheres–water and a lead-shot–water system and for bottom-restrained nonfed, or batch, fluidization of the same system.

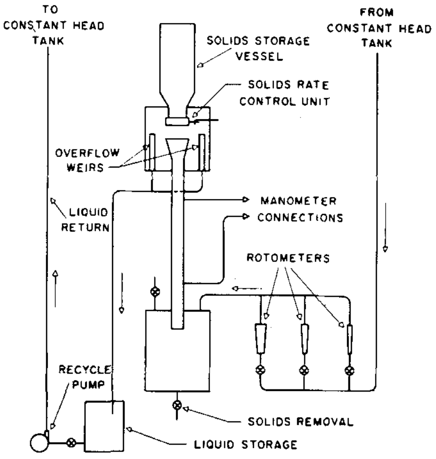


**Fig. 1 Schematic diagram of experimental apparatus.**

The studies were carried out in towers of ¾- to 1.5-in. diameter with glass spheres 0.0183 and 0.0323 in. in diameter and lead shot 0.0500 in. in diameter fluidized by water. Measurements were made of the pressure drop across the fluid bed and of the fluid and particle velocities for various combinations of the physical variables.

The experimental data for countercurrent flow are in excellent agreement with an operating diagram (a plot of holdup vs. fluid velocity with particle velocity as the system parameter) predicted theoretically from experimentally determined batch fluidization data for the same system. Flooding of the experimental systems is also shown to be predictable on the basis of the same batch fluidization data. Thus for the types of operation studied the experimental results support the validity of a single characteristic holdup vs. slip-velocity curve for an ideal moving fluidized system.

TABLE 1. DESCRIPTION OF TOWERS USED

| | I.D., in. | Length, in. | Pressure taps | Conical sections |
|---|---|---|---|---|
| | | | Tower 1 | |
| Type A | 0.653 | 58.4 | One tap 12 in. from bottom; other tap 36 in. higher | Cone angle of 16 deg.; height of 5.0 in. |
| Type B | Section 4.56 in. long, 0.653 in. I.D. added to bottom of type A; tapered gradually to an I.D. of 1.00 in. | | | |
| Type C | Section 4.56 in. long, 0.653 in. I.D. added to bottom of type A. | | | |
| | | | Tower 2 | |
| Type A | 0.970 | 58.4 | One tap 10 in. from bottom; other tap 41.75 in. higher | Cone angle of 17 deg.; height of 4.0 in. |
| Type B | Section 6.17 in. long added to bottom of type A, tapered gradually to an I.D. of 1.88 in. | | | |
| | | | Tower 3 | |
| Type A | 1.50 | 58.4 | Tap 10 in. from each end | Cone angle of 13 deg.; height of 5.0 in. |
| Type B | Section 6.0 in. long added to bottom of type A; tapered to I.D. of 2.44 in. | | | |

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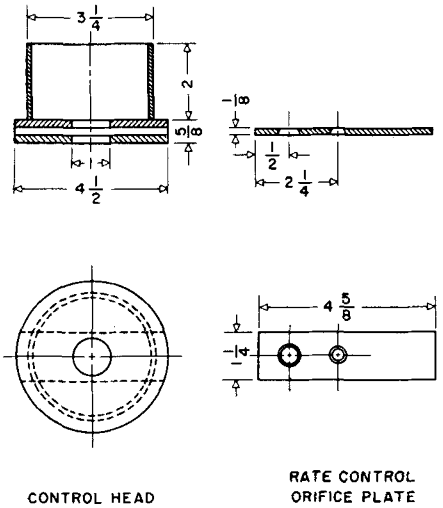


Fig. 2 Solid feed-rate control unit.

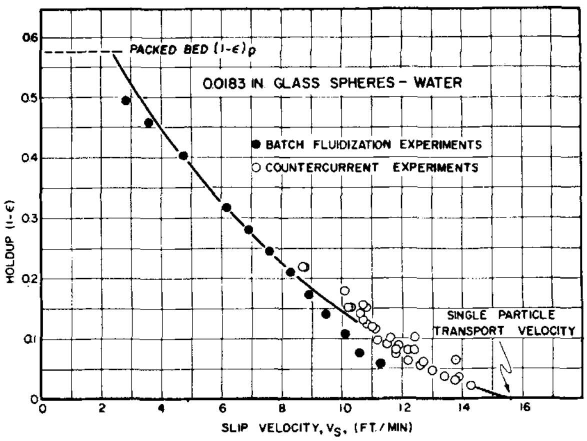


Fig. 3 Experimental holdup ($1 - \epsilon$) vs. slip velocity for 0.0183-in. glass spheres in water

## APPARATUS AND PROCEDURE

### Countercurrent Flow of Fluid and Solids

The main sections of the apparatus were three interchangeable towers of ¾-, 1-, and 1.5-in. nominal diameter borosilicate glass, a constant-head tank for storing the continuous phase, and a storage vessel for holding the amount of solids used during any one experimental run. In addition, provisions were made for equipment to control the rate of solids flow, circulate the fluid through the column, measure the pressure differential across the column, and dry the solids at the conclusion of each run. Figure 1 is a schematic diagram of the over-all equipment. Table 1 presents the dimensions and other pertinent data for the three basic towers and modifications of these.

### Solids Properties

Three different spherical solids were used; two were Scotchlite glass beads and the other was lead shot. A random sample of one hundred and fifty of the larger size glass beads had an average diameter of 0.0323 in. and an absolute density of 177.6 lb./cu. ft. Approximately the same number of the smaller glass beads had an average diameter of 0.0183 in. and an absolute density of 180.4 lb./cu. ft. The lead spheres had an average diameter of 0.0500 in. and an absolute density of 695.6 lb./cu. ft.

### Solids Rate-Control Mechanism

Figure 2 shows the construction details of the unit which controlled the rate of solids feed to the tower. The unit was attached to the base of the storage container. By moving the orifice plate back and forth two ranges of solids flow could be obtained. The fabrication of a number of these orifice plates with different-size orifice holes was used to obtain any predetermined solids rate.

During an experimental run the path of solids flow was from the solids control unit, through the air-water interface, down through the tower, and finally into the lower receiving section. The continuous phase, water, was passed upward through the tower countercurrent to the direction of solids flow. Water flowed in a closed circuit through the apparatus.

### Procedure for Countercurrent Operation

The principal steps in the procedure were, in sequence, the determination of the weight rate of solids flow; the control and measurement of the fluid flow rate; the recording of manometer interface displacements, which indicated the pressure differential in the tower; the establishment of the flooding point; and the drying of the solids preparatory to the next run.

After calibration of the orifice to be used during the run, the solids container was placed in position above the upper receiving section and fully charged with the solids. The tower and both receiving sections were filled with water. The solids flow was then started by sliding the orifice plate into the correct position and initiating and adjusting the fluid flow to the rate desired.

The rotameter float position was recorded along with the associated manometer readings after the latter had remained constant for a minute or more. The process of increasing the fluid flow rate and recording the accompanying manometer readings, after equilibrium conditions had been attained, continued until a small change in rate of fluid flow resulted in the formation of a small (½- to ¾-in.), stable, relatively dense head of solids at the junction of the conical entrance section and the constant-diameter portion of the tower. The tower was then, by definition, flooded. In only a relatively small number of runs was the investigation of pressure drop as a function of fluid flow rate carried beyond the flooding point. After the data at flooding had been recorded, the solids and fluid flow were stopped and the accumulated solids drained and dried.

### Unfed Bottom-Supported, or Batch Fluidization

Three columns, ¾-, 1.0-, and 1.5-in. nominal diameter by 40 in. long, were used to study the fluid-velocity–fraction-void relationship present during screen-supported fluidization of the same fluid-solids systems used in countercurrent flow. The beds were supported by bronze metal screens with, approximately 0.4-mm. openings, below which there were no calming sections. Some check runs were made with the glass beads in a 2- to 4-in. packed bed of 0.050-in. lead spheres between the screens and the glass beads.

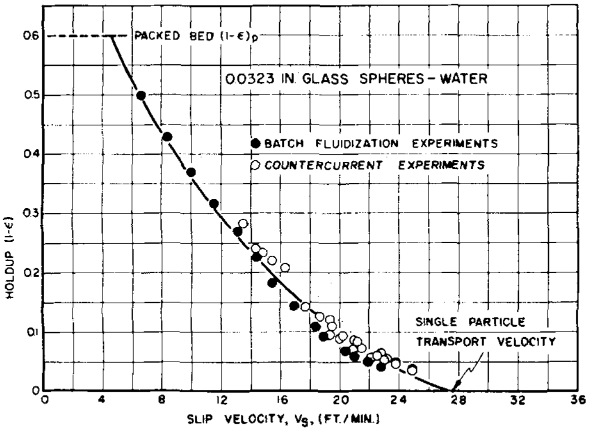


Fig. 4 Experimental holdup ($1 - \epsilon$) vs. slip velocity for 0.0323-in. glass spheres in water.

## THEORETICAL SUMMARY

In this section a brief résumé will be presented of the salient features of the theory previously presented (*1*) for ideal fluidized systems and then applied specifically to free countercurrent operation of fluid-particle systems. Certain basic concepts and assumptions are necessary. These may be listed as follows:

1. Any mass of particles, whether solid, liquid, or gaseous, suspended in a continuous fluid because of the motion of the fluid between the particles is a fluidized system governed by the same fundamental laws.

2. The behavior of any vertical fluidized system in the steady state is determined by the slip velocity between fluid and particle, defined as the vector difference between fluid and particle velocities. Mathematically this may be written as

$$V_s = V_f - V_d = \frac{V_f'}{\epsilon} - \frac{V_d'}{1 - \epsilon} \quad (1)$$

Velocities are taken as positive in the upward direction and negative in the downward direction.

3. For any mass of particles and any fluid of specified properties the relationship between void fraction and the slip velocity is the same irrespective of their direction of motion relative to each other or to the walls; that is, at the same value of the slip velocity all such systems will have the same holdup. This may be expressed by

$$V_s = \phi(1 - \epsilon) \quad (2)$$

4. The basic properties may be generalized in terms of goemetry (size and shape), the physical properties of the system, and the flow rates, regardless of whether the particles are solid, liquid, or gaseous.

Two other quantities and relationships are of interest: the pressure gradient across the fluid bed and the specific contact surface of the particles. These are expressible mathematically by the following two relations for spherical particles:

$$\Delta P/Z = (1 - \epsilon)(\rho_d - \rho_f) \quad (3)$$

and

$$\bar{a} = \frac{6(1 - \epsilon)}{D_p} \quad (4)$$

It is important to note that the holdup, the pressure drop, and the surface area are directly proportional to each other.

In terms of these quantities it may be shown that all vertical moving fluidized systems can be organized into simple standard classes. One can recognize two general classes: a *free* system in which no obstruction or restraint to net forward particle movement exists in the path of flow, type A, and the *restrained* system, in which the forward particle movement but not the fluid is restrained mechanically as with a screen or membrane nonpermeable to the particles, type B. These two general classes can each be subdivided into a number of standard types depending upon the direction of flow of the two phases. One of these types is countercurrent flow, in which the solids move downward through a rising continuous fluid.

By combining Equations (1) and (2) either graphically or analytically, a generalized operating diagram can be constructed in which the holdup or pressure drop is plotted vs. the fluid velocity with solids velocity as a parameter. The behavior of all the classes and types of fluidized systems can be shown on this diagram. A portion of it will be considered later. In free countercurrent operation with a fixed particle flow rate there exists an upper limit on the fluid velocity or the holdup. At this upper limit, referred to as the flooding point, the system will begin to reject the particle feed. On the generalized operating diagram the locus of all the flooding points is shown and indicates the highest holdup attainable in a free system at a specified solids rate.

At the present time the function of Equation (2) may best be determined experimentally by using the batch expansion of an unfed screen-supported bed. In this case the net particle velocity with respect to the walls is arbitrarily made zero; $V_d = 0$. Thus the slip velocity equals the fluid velocity, and Equation (2) can be determined immediately. It is also possible to predict Equation (2) from empirical correlations in the literature (*2*, *3*, *4*, *8*). In general these involve plotting the variables included in any one of several possible combinations of dimensionless groups. The present correlations need to be improved and extended over wider ranges of physical properties and Reynolds number before they can be used with complete confidence.

## RESULTS AND DISCUSSION

### The Invariant-Holdup–Slip-Velocity Relationship

*Glass-Spheres–Water System*

If the theory and postulates proposed are correct, the holdup–slip-velocity curve should be the same irrespective of the mode of operation of the moving fluidized system. Thus the data for the free countercurrent system and the batch or unfed screen-supported system should coincide. Figures 3 and 4 represent the experimental comparison for the two sizes of glass beads. The slip velocity was calculated by the use of Equation (1)

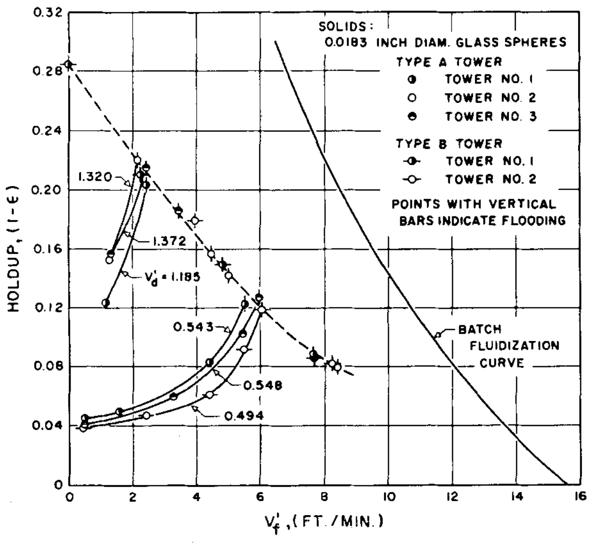


Fig. 5 Experimental holdup $(1 - \epsilon)$ vs. superficial fluid velocity for 0.0183-in. glass spheres in water, superficial solids velocity as parameter.

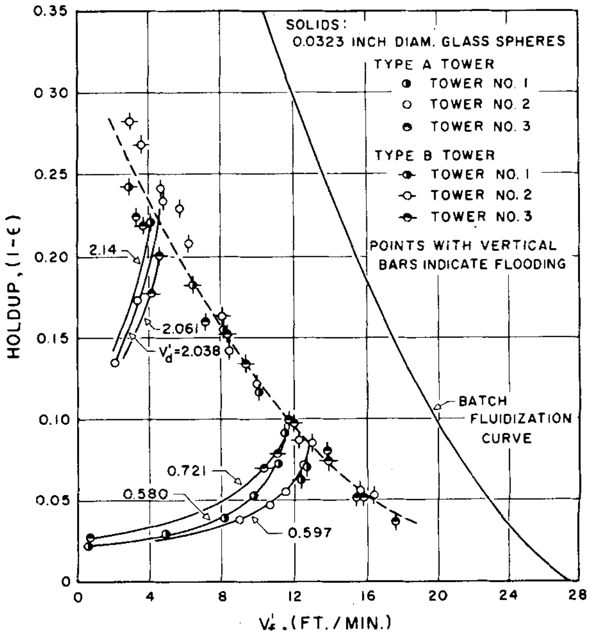


Fig. 6 Experimental holdup $(1 - \epsilon)$ vs. superficial fluid velocity for 0.0323-in. glass spheres in water, superficial solids velocity as parameter.

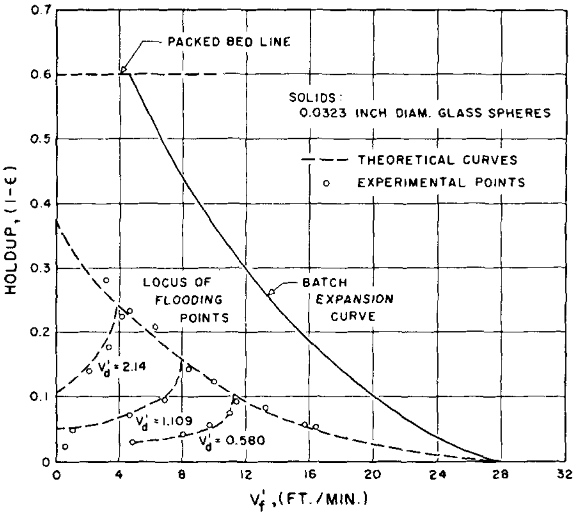


Fig. 7 Experimental vs. theoretically predicted generalized operating diagram for 0.0323-in. glass spheres.

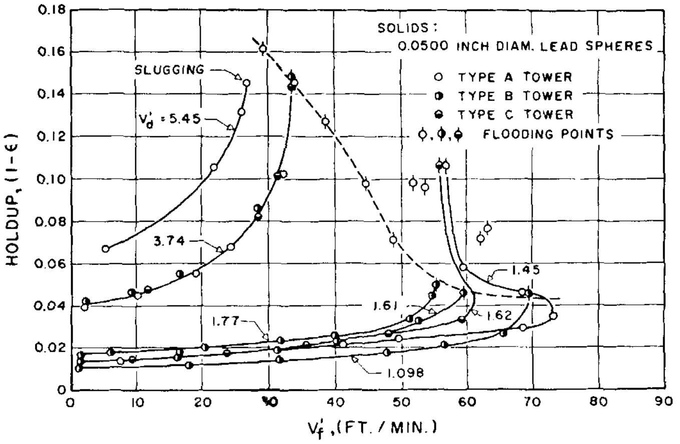


**Fig. 8 Experimental holdup ($1 - \epsilon$) vs. superficial fluid velocity for 0.0500-in. lead shot in water, superficial solids velocity as parameter.**

and the holdup by the use of Equation (3) in conjunction with the measured values of the pressure drop. The data represent a very wide range of solids and fluid feed rates. The original data may be found in Price's thesis (7).

The solid line representing the batch fluidization data extends from the left at the packed-bed condition to the right at the single-particle transport velocity. One notes that the agreement between the two types of operation on each graph is excellent. The data illustrated for the countercurrent systems represent only a portion of these collected, but excessive crowding of the points would result if further data were used. The velocity corresponding to zero holdup, that is the single-particle transport velocity, was calculated by the commonly used drag coefficient (6). It is interesting to note that the maximum holdup in free countercurrent operation is limited to approximately 0.22 for the 0.0183-in. glass beads and 0.28 for the 0.0323-in. glass beads, whereas the packed bed corresponds to a void fraction of 0.58 and 0.60 respectively. Thus, as may be predicted theoretically, flooding of the free countercurrent system occurs at moderately low holdups. The data for the 0.0500-in. lead shot fluidized by water exhibit essentially the same behavior. Thus the validity of the single-invariant holdup–slip-velocity curve for any one system, as indicated by Equation (2), would seem to be valid for the types of operations and systems herein considered. The recent work of Mertes and Rhodes (5) in which measurements were made on countercurrent systems has also shown the same dependence of slip velocity on holdup.

#### Comparison with Predicted Operating Diagram

Figures 5 and 6 are plots of the holdup as a function of $V_f'$ for the free countercurrent flow of water and 0.0183- and 0.0323-in. glass spheres, respectively. The parameter for the curves on both figures is a constant $V_d'$. Also shown on each figure is the corresponding batch expansion data. Three different tower diameters each with two different end arrangements are represented. The dashed line represents the best line through the flooding points.

The rate of change of holdup as a function of liquid flow rate was similar for both sizes of glass beads. At low liquid rates the holdup changed quite slowly, but, as the flooding point was approached, the holdup was rapidly increased by only small changes in fluid rate.

At low values of holdup the solids-liquid mixture appeared to be homogeneous; however at relatively high holdups small-scale heterogeneity was evident. With a strong source of illumination behind the column the solids appeared to be falling in thin strata. The use of a stroboscope indicated that there was no regular interval of repetition associated with the formation of these layers.

#### *The Flooding Locus Curve*

In Figures 5 and 6 any point below the locus of the flooding curve and $V_f' = 0$ represents stable free countercurrent operation. Any point between the batch expansion curve and the locus of the flooding curve represents stable, restrained countercurrent operation. By definition, at flooding the continuous and discontinuous phase rates are such that a slight increase in either will cause the formation of a head of the discontinuous phase in its conical entrance section, in other words the rejection of a portion of the feed of discontinuous particles from the effective zone of the tower. With towers having a constant diameter throughout, flooding at a fixed solids rate occurs at the maximum fluid velocity that will allow the tower to accept all the solids being fed. An increase in the rate of either phase causes rejection of part of the solids feed at the point of its entrance to the tower.

If a system is operated at any point below the flooding point and the fluid velocity is gradually increased, the holdup will increase along a $V_d' =$ constant curve until flooding of the system occurs. Hence flooding in a free countercurrent system corresponds to the maximum holdup possible in a constant-diameter tower at any constant solids rate. In addition the flooding point may be represented by the equation

$$\left.\frac{\partial V_f'}{\partial(1 - \epsilon)}\right|_{V_d'} = 0 \qquad (5)$$

The rejection of solids increases as $V_f'$ is further increased, and the solids holdup decreases until finally the single-particle terminal velocity is reached.

In general there was no effect of column diameter during either the batch or free countercurrent operation of the glass-spheres fluidized systems. As might further be expected, the locus of the flooding curve was not affected by the column diameter employed.

As has been previously indicated, the use of Equations (1) and (2) combined with data from a single-batch unfed bottom-supported fluidization experiment allows one to illustrate graphically the behavior of all moving fluidized systems. This plot is termed the *generalized operating diagram.* Since there are three parameters involved, a number of different types of plots can be constructed, but the authors find that holdup vs. fluid velocity with solids velocity as the parameter is the most useful and convenient. Figure 7 represents such a diagram for the 0.0323-in. glass beads. The method used in constructing Figure 7 has been described in detail (1) and consists of a graphical procedure. It is possible of course to represent Equation (2) by an empirical least-squares polynomial fitted to the batch fluidization data. Under these conditions the graphical procedure for constructing the operating diagram can be dispensed with and an analytical procedure used. This latter technique will be illustrated in detail in subsequent publications.

The dashed lines on Figure 7 represent the theoretically predicted behavior of the free countercurrent system, and the points represent the corresponding experimentally determined values. Only a few sets of experimental points are shown in order not to crowd the plot. In general the agreement between the predicted and the experimental is quite good. The maximum in the holdup–fluid-rate curve for this free countercurrent flow, measured experimentally, corresponds to the locus of the flooding points as determined visually by rejection of solid particles. This experimentally observed curve also corresponds closely to the one predicted from the theory and batch expansion data by the methods outlined (Figure 7).

*Lead-Shot–Water System*

Whereas the data for the glass-spheres–water system were found to be independent of the physical dimensions and construction of the towers, this was not true for the lead-shot–water system. In the latter case the construction of the solids outlet section of the tower proved to be important, especially at low solids rates. Figure 8 is a plot of the data for the 0.0500-in. lead shot flowing countercurrently to water in the three different modifications of tower 1. (See Table 1.)

At high and intermediate values of the solids rate the holdup vs. fluid velocity curves were independent of the tower used, and the curves exhibited the type of behavior predicted. At the flooding point for these solid rates a small, very dense, highly turbulent layer of shot was formed in the conical entrance directly above the constant-diameter section of the tower. Another very small increase in liquid rate caused the solids head to continue to build up until, at approximately 4.5 in., bridging of the solids occurred at the junction of the cone and tower proper. The design of the approach angle of the cone proved to be very important in determining when this bridging would occur. During the interval of solids build-up in the cone a fixed volume of the head was fluidized, and the remainder of the particles acted as a slowly moving, fully packed bed. The shot were fed into the tower proper only after falling on top of the solids head in the cone and moved slowly downward through the fully packed region into the fluidized zone.

At low solids and liquid rates extremely low holdups were obtained with a visible solids concentration gradient in the radial direction. All but a few of the particles fell in the region adjacent to the tower wall, thereby leaving the central core of the tower almost free of solids. At the base of the tower, on the other hand, the solids distribution appeared to be uniform. As the fluid velocity increased in tower type A, there was *channeling* of the solids and liquid at the tower base. Relatively dense groups of particles were observed to by-pass regions of low particle concentration with the resulting formation of swirls and eddies within the solids-liquid mixture with a scale approximately equal to the tower diameter. At some higher fluid velocity there suddenly appeared at the tower base a small region of very high holdup which spread rapidly up the tower. As the average holdup and resulting pressure drop increased, the fluid flow decreased, and a stable condition was attained where the lower half of the tower was filled with a very dense phase and the upper half with a much more dilute one. A small increase in the liquid rate caused the dense phase to move up the tower and into the cone. This lead-shot phenomenon is due to the creation of a partially restrained bed at the solids exit and produces the high holdups. The flooding in this case is not true flooding but rather an expansion of this restrained bed to overflow the top of the tower when the rate of withdrawal at the bottom does not equal the solids feed rate; in other words this is not truly a free countercurrent system.

By the addition of a flaired section to the tower's base (B type of modification) the phenomenon of dense-phase formation was eliminated (Figure 8). There was visual evidence that the tendency to form a dense solids layer at the lower end of the tower porper was still present, but the flaired section reduced the tendency sufficiently so that flooding was reached before any such region of high holdup formed. The removal of the flaired section and the addition of one having a constant diameter equal to that of the tower proper (C type of modification) gave results identical to those obtained in the original tower.

In this system, with a large difference between the density of particles and fluid, one begins to observe deviations from ideal or particulate fluidization. Since the basic theoretical postulates of the present work assume ideal fluidization, it is not surprising that a certain degree of deviation from the theoretical behavior occurs. It is very encouraging however to note that with the proper experimental equipment even these systems behave as predicted.

## CONCLUSIONS

The agreement found between the holdup–slip-velocity relations for free countercurrent flow and for batch fluidization supports the basic assumption of the existence of a single fundamental relationship between holdup and slip velocity for any vertical fluidized system. This indicates that all vertical fluidized systems may be interrelated and generalized in terms of slip velocity and void fraction irrespective of the direction of motion. It also has been shown that the theory developed in a previous publication allows an accurate prediction of the operation of free countercurrent systems. The method of prediction requires data from a single-batch unfed bottom-supported fluidized experiment.

It has been verified experimentally that in free countercurrent flow there exists a maximum in holdup–fluid-rate curves, which corresponds to the flooding point of the system, as indicated by visual observation of the rejection of solid particles, and to the values predicted from batch fluidization data by the theory presented.

## NOTATION

$V_s$ = slip velocity, ft./min.
$V_f$ = continuous fluid average velocity, ft./min.
$V_d$ = discontinuous or solids average velocity, ft./min.
$V_f'$ = superficial continuous fluid velocity, ft./min.
$V_d'$ = superficial discontinuous or solids velocity, ft./min.
$1 - \epsilon$ = fraction holdup of solids in tower
$\epsilon$ = void fraction of tower
$\Delta P/Z$ = static pressure drop, lb./(sq. ft.) (ft.) of length
$\rho_f$ = density of continuous phase, lb./cu. ft.
$\rho_d$ = density of discontinuous phase, lb./cu. ft.
$\bar{a}$ = specific contact area of solids/unit volume
$D_p$ = solids diameter
$\phi$ = slip velocity functionality on $(1 - \epsilon)$

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